

Statistical Entropy of Near Extremal Five-Branes

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Abstract

The Hawking Beckenstein entropy of near extremal fivebranes is calculated in terms of a gas of strings living on the fivebrane. These fivebranes can also be viewed as near extremal black holes in five dimensions.

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Recently the Bekenstein-Hawking entropy of certain extremal BPS black holes was precisely calculated by counting microstates in string theory [1-4]. Since black holes are non-perturbative objects, the calculations required considerable understanding of non-perturbative string theory and a certain class of solitons called D-branes [5].

These results were extended to leading order away from extremality in [6-9]. These calculations are not so well justified as the BPS ones because one naively expects strong coupling effects in the semiclassical regime in which the black hole picture is valid. Nevertheless it was argued that strong coupling effects could be avoided in certain corners of the parameter space sufficiently near to extremality. The surprising agreement discovered in [6-9] between the string and black hole calculations indicate that under favorable circumstances strong coupling effects could be avoided. The calculation in this paper suffers a similar kind of strong coupling problem, but it extends the ones in [6-9] to a wider class of black holes, finding an interesting interpretation of the classical formulas.

In [10,11] the entropy of some near extremal black p -branes was observed to scale as that of a gas of particles living on the brane. The ten dimensional fivebrane that we consider here is a different case which did not fit in the analysis of [11]. In [12] the entropy for BPS fivebranes was calculated in terms of strings living on the fivebrane, here we try to give the description for non-BPS configurations.

1. Classical solutions

In ref. [13] general non extremal black hole solutions to type II supergravity compactified on $T^5 = T^4 \times S_1$ were considered. These black holes are characterized by three independent charges, their mass and two sizes of the compact space, the radius R of S_1 and the volume $(2\pi)^4 V$ of T^4 . The three charges correspond to the number Q_5 of D-fivebranes wrapped on T^5 , the number Q_1 of D-strings wrapped on S_1 and the momentum N/R flowing along S_1 . We use the notations of [13]. We will be considering the limit V, γ large and r_0 small with $V r_0^2$ fixed and $r_0^2 e^{2\gamma}$ fixed. In this limit the black hole is more properly interpreted as a black fivebrane in ten dimensions if the radius R is also large or as a black four-brane in nine extended dimensions if the radius R is small. Alternatively we could think that it is a near extremal black hole in five dimensions with only one charge, the other two charges would be zero or small, of the order of the deviation from extremality. U-duality [14] maps this solution into the solutions describing the near extremal limit of black holes coming from D-branes wrapped on T^5 , or solitonic fivebranes wrapped on

T^5 or fundamental strings winding along T^5 , in summary, the near extremal limit in five extended dimensions of all BPS black holes that preserve 1/2 of the supersymmetries.

In this limit the mass of the black hole becomes [13]

$$M_{BH} = \frac{RVQ_5}{g\alpha'^3} + \frac{RVr_0^2}{2g^2\alpha'^4}(\cosh 2\alpha + \cosh 2\sigma) = M_{bare} + M , \quad (1.1)$$

where the first term is interpreted as the mass of the bare extremal D-fivebranes. The entropy reads, in this limit,

$$S = \frac{A_{10}}{4G_{10}} = \frac{A_5}{4G_5} = \frac{2\pi RVr_0^2}{g^{3/2}\alpha'^{7/2}}\sqrt{Q_5}\cosh\alpha\cosh\sigma , \quad (1.2)$$

and the winding (of 1D branes) and momentum charges are [13]

$$\begin{aligned} Q_1 &= \frac{Vr_0^2}{2g\alpha'^3}\sinh 2\alpha, \\ n &= \frac{R^2Vr_0^2}{2g^2\alpha'^4}\sinh 2\sigma . \end{aligned} \quad (1.3)$$

It was shown in [13] that the classical black hole Hawking Beckenstein entropy has the simple form

$$S = 2\pi(\sqrt{N_1} + \sqrt{N_{\bar{1}}})(\sqrt{N_5} + \sqrt{N_{\bar{5}}})(\sqrt{n_L} + \sqrt{n_R}) \quad (1.4)$$

in terms of a collection of $(N_1, N_{\bar{1}}, N_5, N_{\bar{5}}, n_R, n_L)$ non-interacting branes, anti-branes and strings. For notations see ref. [13]. The limit we are considering corresponds to $N_{\bar{5}} = 0$, while the limits previously considered involved two of the anti-charges very small [7,13] or all anti-charges small [6].

In the case that the two charges (1.3) are small ($\alpha \sim \sigma \sim 0$) the Hawking temperature of the near extremal solution is constant with the value

$$T_H^{-1} = 2\pi\sqrt{g\alpha'Q_5} . \quad (1.5)$$

When the deviation from extremality becomes very small the classical thermodynamic description would break down and we expect that the physical temperature would drop to zero. One would estimate that this happens when the mass above extremality becomes of the order of the Hawking temperature $M \sim T_H$ (1.5).

General rotating black holes in five dimensions were constructed in ref. [15]. In the limit we are considering, their entropy formula becomes

$$S = 2\pi \sqrt{Q_5 \frac{r_0^4 V^2 R^2}{4g^3 \alpha'^7} (\cosh \alpha \cosh \sigma + \sinh \alpha \sinh \sigma)^2 - \frac{(J_1 + J_2)^2}{4}} + \\ + 2\pi \sqrt{Q_5 \frac{r_0^4 V^2 R^2}{4g^3 \alpha'^7} (\cosh \alpha \cosh \sigma - \sinh \alpha \sinh \sigma)^2 - \frac{(J_1 - J_2)^2}{4}}. \quad (1.6)$$

2. Effective strings

Before we say anything about D-branes let us calculate the energy and entropy that a gas of strings living on the fivebrane would have. We consider superstrings with tension $T_{eff} = \frac{1}{2\pi\alpha'_{eff}}$ and an effective number of degrees of freedom c_{eff} , where $c_{eff} = N_B + \frac{1}{2}N_f$ is the effective central charge in the static gauge, which we assume to be the same for left and right movers, since the central charge is not 12 there are potential problems for constructing a superstring theory. For the moment we will work in this gauge and we will not worry about the problems with Lorentz covariance that might arise. All we will do now is to rewrite the entropy formula in a suggestive way, inspired in this effective string idea. Without much justification we will take this string to be non interacting. We take this gas to have total mass M , and some net momentum and winding (n, m) along the S^1 direction. The entropy of this gas comes mainly from a configuration in which we have just a single long string. The right and left moving momenta of this long string in the S^1 direction are

$$p_{R,L} = \frac{n}{R} \pm \frac{mR}{\alpha'_{eff}}. \quad (2.1)$$

The Virasoro constraints are

$$M^2 = p_R^2 + \frac{4}{\alpha'_{eff}} N_R, \quad M^2 = p_L^2 + \frac{4}{\alpha'_{eff}} N_L. \quad (2.2)$$

where N_L, N_R are the left and right total oscillator levels. Using the number of physical degrees of freedom that we mentioned above, the entropy becomes

$$S = 2\pi \sqrt{c_{eff}/6} \left(\sqrt{N_L} + \sqrt{N_R} \right). \quad (2.3)$$

Let us identify the mass of this string gas with the the second term in the mass of the black hole (1.1), the momentum n with the charge n in (1.3) and the the winding number m with the charge Q_1 as

$$Q_1 = m \frac{g\alpha'}{\alpha'_{eff}} \quad (2.4)$$

which follows from comparing the black hole BPS mass formula and (2.2) for strings with pure winding. Now, using (2.2)(2.1) we can calculate $N_{R,L}$

$$N_{R,L} = \frac{r_0^4 V^2 R^2}{4g^3 \alpha'^7} \frac{\alpha'_{eff}}{g\alpha'} (\cosh \alpha \cosh \sigma \pm \sinh \alpha \sinh \sigma)^2 \quad (2.5)$$

so that (2.3) would reproduce (1.2) if

$$\frac{c_{eff}}{6} \cdot \frac{\alpha'_{eff}}{g\alpha'} = Q_5 . \quad (2.6)$$

The Hagedorn temperature for this string is $T_H^{-1} = 2\pi\sqrt{\alpha'_{eff}c_{eff}/6}$ which, using (2.6) becomes (1.5).

Note that all we have done in this section is to make a change of variables and rewrite the entropy formula (1.2) in the suggestive form (2.3). The entropy presented in this way strongly suggests a microscopic description in terms of strings living on the fivebrane. This gives a very natural explanation for the constant temperature (1.5).

3. D-brane picture

We will now show that strings obeying (2.4)(2.6) arise naturally in the D-brane description of black holes. In the D-brane picture, extremal black holes can be viewed as a collection of Q_5 D-fivebranes wrapping on $T^4 \times S^1$, Q_1 D-strings wound along S^1 with some momentum N flowing along the direction of the strings [1,6]. In [16] the D-strings were pictured as instantons in the $U(Q_5)$ gauge theory of the fivebrane, more precisely, the D-strings are the zero size limit of these instantons. In [17] it was argued that the moduli space of one of these instantons corresponds to an $N = 4$ superconformal field theory with $4Q_5$ bosonic and $4Q_5$ fermionic degrees of freedom. So we can say that they are “instanton strings” with $4Q_5$ bosonic and $4Q_5$ fermionic physical oscillators. The same conclusion can be achieved with the analysis in [6] where this number of degrees of freedom is related to open strings going between the D-string and the Q_5 D-fivebranes. We are now concentrating in just one D-string at a time and we are considering only the massless modes of the D-string. The inverse tension of this string is just that of the D-string $\alpha'_{eff} = g\alpha'$, and the effective central charge is $c_{eff} = 6Q_5$ so that (2.4)(2.6) are satisfied. One puzzling aspect of this picture is that the string tension is quite large for weak coupling so that one might wonder why just a gas of fundamental strings is not giving a bigger contribution. One answer to this is to note that for large Q_5 , and energies such that $M_{bare} \gg M \gg \frac{1}{\sqrt{g\alpha'}}$, these

“instanton strings” have much more entropy. This suggests that the entropy formula (2.3) will be valid only for energies bigger than $M \sim \frac{1}{\sqrt{g\alpha'}}$. However, from the classical black hole theory we would expect it to break down only when $M \sim T_H$ (1.5). These two scales are very different for large Q_5 . In [18] similar problems were solved by proposing that, due to the way the branes are wrapped, the effective quantum of momentum for excitations moving along the compact direction is changed $1/R \rightarrow 1/RQ_5$. In a similar fashion we expect that a more correct picture should be that these “instanton strings” break into Q_5 fractional instantons with instanton number $1/Q_5$. Indeed it was found in [19] that there are solutions of $U(Q_5)$ gauge theories on a torus with fractional instanton number $1/Q_5$. These fractional strings would have inverse string tension $\alpha'_{eff} = Q_5 g\alpha'$ and $c_{eff} = 6$ so that (2.4)(2.6) continues to be satisfied². A picture of strings with renormalized tension was advocated in [21] to account for the entropy in the BPS case. These strings loosely resemble those proposed by Susskind [22]. Of course there is another problem which is that of strong coupling. It is apparently present in all discussions of non-extremal black holes and makes these formulas look even more puzzling.

As a further check on the effective string picture we will calculate the entropy for the case with angular momentum in the extended 1+4 dimensional space. The worldsheet theory, in the static gauge, of these “instanton strings” is an $N=(4,4)$ superconformal field theory. It was argued in [2,8] that the angular momentum is carried by the fermions of the $N=(4,4)$ superconformal field theory. More precisely, the $N=(4,4)$ algebra has a $SU(2)_R \times SU(2)_L$ R-symmetry which, as argued in [2], is linked to the decomposition of the spatial rotations as $SO(4) = SU(2)_R \times SU(2)_L$. In $SO(4)$ language the angular momentum is characterized by two eigenvalues J_1, J_2 corresponding to the angular momenta on two orthogonal planes and, in terms of the $U(1)$ charges (F_R, F_L) of the $N = 4$ algebra, they read [2]

$$J_1 = \frac{1}{2}(F_R + F_L) , \quad J_2 = \frac{1}{2}(F_R - F_L) . \quad (3.1)$$

Using the same arguments as [2,8] we find that the net effect will be to replace in (2.3)

$$N_L \rightarrow \tilde{N}_L = N_L - \frac{(J_1 - J_2)^2}{2c_{eff}/3} , \quad N_R \rightarrow \tilde{N}_R = N_R - \frac{(J_1 + J_2)^2}{2c_{eff}/3} \quad (3.2)$$

² It would be very interesting to find the relation between these effective strings and the ones in [12][20].

so that

$$\begin{aligned}
S &= 2\pi \sqrt{\frac{c_{eff}}{6}} \left(\sqrt{\tilde{N}_R} + \sqrt{\tilde{N}_L} \right) \\
&= 2\pi \sqrt{\frac{c_{eff}}{6} N_R - \frac{(J_1 + J_2)^2}{4}} + 2\pi \sqrt{\frac{c_{eff}}{6} N_L - \frac{(J_1 - J_2)^2}{4}},
\end{aligned} \tag{3.3}$$

which, using (2.5),(2.6) reduces to (1.6).

4. Discussion

In summary, a remarkable formula (2.3) has been found for the entropy on near extremal fivebranes using a D-brane inspired picture in terms of strings living on the fivebrane. The near extremal limit considered here is more generic than the ones considered in [7,13,8,6], unfortunately the objects we are dealing with are not so well defined. The energy dependence of the near extremal entropy is well reproduced by a gas of strings. The precise numerical coefficient requires a more specific property (2.6) of this effective string theory. This property is precisely obeyed by “instanton strings” arising in the D-fivebrane gauge theory. In the PBS limit these strings were used to explain the entropy [1]. It will be interesting to study more precisely these strings in the spirit of [20,12], we see that some more information can be gained by studying near extremal black holes. Under U-duality the D-fivebrane becomes a solitonic fivebrane and the D-strings fundamental strings. It will be interesting to connect the present picture with the dilaton gravity studies [23].

It is very interesting that the near extremal black hole entropy can be viewed as the entropy as a gas of particles when two of the charges are large [7] and as the entropy of a gas of strings when only one of the charges is large. The general case, which includes the Schwarzschild black hole, might involve also strings or some other extended object.

It seems straightforward to generalize these arguments to four dimensions using the results in [3,9] to the case where two of the anti-charges are different from zero. These black holes are U-dual to the near extremal limit of Sen’s electrically charged black holes [24]. Indeed the thermodynamics features of [24] suggest an effective string picture since the near extremal Hawking temperature of Sen’s black holes is constant (therefore the entropy is linear in the mass difference $M_{BH} - M_{extr}$).

A word of caution should be said: we have overlooked two problems, one is how to get a fully consistent free superstring theory and the other is how they manage to be weakly coupled. Hopefully the resolution to these problems will lead to a more precise picture. It

is however very interesting to have found a black hole where the entropy is indeed given by a gas of strings.

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